SEPD 4.0: Self-Entangled Polar-Dual Dynamics Polar-Dual Topology with Bicrossproduct Computational Proxy

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1 Special Preliminary Explanation

No rigorous self-contained mathematical framework corresponding to the polar-dual product $\circledast$ (ontological self-coupling of polar substrate) has been constructed so far. All algebraic derivations, spectral calculations and representation theory in this paper adopt Majid's bicrossproduct structure as an approximate mathematical carrier to characterize the dual-phase manifestation behavior of the primitive polar substrate. The bicrossproduct serves only as a calculational substitute, and there exists an irreducible conceptual gap between binary bicrossproduct algebra and monistic polar-dual ontology, which is fully marked and discussed in the full text.

Abstract

We establish the complete theoretical framework of SEPD 4.0 Self-Entangled Polar-Dual Dynamics based on polar-dual topology. The core ontological primitive is the polar-dual product $\circledast$, which labels the intrinsic self-entanglement of the unique primitive polar substrate $\mathcal{U}$ and satisfies the fixed-point identity $\mathcal{U} \circledast \mathcal{U} = \mathcal{U}$. Discrete quantum evolution and continuous spacetime geometry are not two pre-independent objects, but two types of dual phases derived from the polar substrate under different observation resolutions. Conventional tensor products, bicrossproducts and other algebraic structures all describe couplings between two separate algebras, which have essential separations from the monistic primitive assumption of "one polar generating two dual phases".

For quantitative deduction, Majid's bicrossproduct $H \bowtie H^\wedge$ is taken as the computational proxy of polar-dual product. We set $H = U_q(\mathfrak{b}_+)$ as the carrier of discrete quantum dual phase and its dual $H^\wedge = U_q(\mathfrak{b}_-)$ to bear continuous geometric dual phase. The deformation parameter $q = |q|e^{i\theta}$ is the intrinsic torsional modulus of the polar substrate. When q takes the primitive 4th root of unity i, the representation theory undergoes a non-semisimple transition, the spectral measure develops square-root singularity, and spontaneous Exceptional Points (EPs) are generated without manual gain-loss tuning, leading to the universal $\sqrt{t}$ transient relaxation law.

Three mutually locked experimental discriminants are provided; violation of any single criterion falsifies the model: relaxation exponent within $[0.48, 0.52]$, vanishing phase rigidity at critical temperature T_c, spontaneous EP formation without external gain-loss modulation. This paper clearly sorts out three inherent gaps between polar-dual product ontology and bicrossproduct mathematics, lists long-term open mathematical problems, and strictly separates ontological discourse and computational derivation without mixing symbols or concealing theoretical limitations.

Polar-Dual Topology; Polar Substrate; Polar-Dual Product; Self-Entanglement; Exceptional Point; $\sqrt{t}$ Relaxation; Bicrossproduct Algebra

2 Introduction

Contemporary quantum gravity universally suffers from a fundamental dichotomy dilemma. Quantum mechanics is built on discrete Hilbert states, while general relativity presupposes continuous Riemannian manifolds. All mainstream theories first define two independent objects and then construct correlations via duality or tensor coupling, failing to simultaneously generate quantum behavior and spacetime geometry from a single primitive entity. String theory presupposes spacetime background before quantizing vibration modes; Loop Quantum Gravity and Causal Sets cut continuous spacetime first and then take continuum limits; Connes' noncommutative geometry matches algebras to known manifolds; conventional non-Hermitian physics requires fine-tuned gain-loss to produce spectral singularities, lacking intrinsic thermodynamic origins for criticality.

The previous SED 1.0, 2.0 and 3.0 series all attempted to bridge the dichotomy gap, but shared common defects: either merely borrowing standard q-algebras without exclusive markers for monistic intuition, or arbitrarily mixing self-defined symbols into all derivation formulas, separating ontological intuition from mathematical computation and being regarded as verbal repackaging of existing quantum group theories.

This work upgrades to SEPD 4.0, formally named *Self-Entangled Polar-Dual Dynamics*, and constructs exclusive terminology: the single primitive foundation is defined as polar substrate $\mathcal{U}$, its internal self-winding interaction is named polar-dual product $\circledast$, and the whole monistic manifestation ontology is collectively called polar-dual topology.

The core writing principle is complete separation of ontology and computation: the symbol $\circledast$ only appears in ontological discussions and never enters any algebraic derivation. All quantitative calculations uniformly adopt mature Majid bicrossproduct as an approximate mathematical substitute. Three irreconcilable conceptual gaps between monistic polar-dual ontology and existing binary algebraic constructions are explicitly marked throughout the text, without splicing various mathematical structures as superficial "shadows".

Notably, all observable predictions are independent of ontological interpretation. Even if the monistic polar-dual hypothesis is discarded, the three experimental criteria ($\sqrt{t}$ relaxation, spontaneous EP, vanishing phase rigidity) remain fully valid only relying on unit-root representation theory of bicrossproduct algebra.

3 Polar-Dual Ontology vs. Bicrossproduct Computational Proxy

3.1 Axiom of Polar Substrate and Polar-Dual Product

Axiom 1 (Primitive Polar Substrate) There exists a unique fundamental polar substrate $\mathcal{U}$. No pre-split discrete algebra or continuous geometric objects exist. The intrinsic self-winding interaction inside the substrate is denoted as polar-dual product $\circledast$, satisfying the fixed-point identity purely for ontological description:

$$\mathcal{U} \circledast \mathcal{U} = \mathcal{U}.$$

Notation Remark: The symbol is a graphical command provided by the `amssymb` package, but this paper assigns an exclusive new semantic definition. All existing literature uses $\circledast$ to represent binary operations between two separate objects; in this work, it exclusively labels the self-entanglement of single polar substrate and carries no binary algebraic meaning.

Four core ontological commitments:

1. Only one primitive polar substrate exists in nature, with no innate dichotomy;
2. Polar-dual product only describes self-interaction of the substrate, not mutual action between two distinct entities;
3. Discrete quantum behavior and continuous spacetime geometry are dual phases appearing at different resolution scales, not fundamental substances;

4. The parameter q is an intrinsic torsional degree of freedom of the polar substrate, not an artificially imported external constant.

3.2 Bicrossproduct as Approximate Calculational Substitute

Majid's bicrossproduct $H \bowtie H^\wedge$ is the closest existing mathematical structure to the polar-dual two-phase picture, yet three irreparable conceptual gaps persist, listed in Table 1.

Table 1: Conceptual Comparison: Bicrossproduct Binary Algebra vs. SEPD Polar-Dual Monistic Ontology

Comparison Dimension	Majid Bicrossproduct	SEPD Polar-Du
Basic Carrier	Two pre-defined independent Hopf algebras $H, H^\wedge$	Single polar substrate; dual pha
Coupling Constraint	Externally imposed matched pair axioms	Self-consistency emerges intrinsically
Parameter q	Artificial algebraic deformation constant	Intrinsic thermodynamic torsion

Three permanent open mathematical gaps:

1. Bicrossproduct pre-separates H and $H^\wedge$, while polar-dual topology forbids any pre-existing binary split;
2. Matched pair conditions are externally attached rules, whereas self-consistent constraints of polar-dual product should fully emerge from internal substrate dynamics;
3. $\bowtie$ is standard binary algebraic operation, while $\circledast$ is merely an ontological marker with no ready-made algebraic definition.

All operator commutation, spectral and representation derivations below adopt standard quantum group symbols $\bowtie, \otimes$; $\circledast$ only appears in ontology-related paragraphs.

4 Bicrossproduct Quantitative Calculation System

4.1 Discrete Quantum Dual Phase: $H = U_q(\mathfrak{b}_+)$

Take Borel subalgebra of quantum group as mathematical proxy for microscopic discrete phase, with generators K, K^{-1}, E satisfying basic commutation:

$$KE = qEK, \quad KK^{-1} = K^{-1}K = \mathbb{1}.$$

Standard coalgebra structure:

$$\Delta(K) = K \otimes K, \quad \Delta(E) = E \otimes K + \mathbb{1} \otimes E.$$

Counit and antipode:

$$\epsilon(K) = 1, \quad \epsilon(E) = 0, \quad S(K) = K^{-1}, \quad S(E) = -EK^{-1}.$$

4.2 Continuous Geometric Dual Phase: Dual Hopf Algebra $H^\wedge = U_q(\mathfrak{b}_-)$

Generator F paired with K obeys inverse deformed commutation relation:

$$KF = q^{-1}FK, \quad \Delta(F) = F \otimes \mathbb{1} + K^{-1} \otimes F, \quad \Delta(K) = K \otimes K.$$

4.3 Bicrossproduct Multiplication and Cross Commutator

General multiplication rule of bicrossproduct algebra:

$$(h \otimes \phi)(h' \otimes \phi') = h \cdot (\phi_{(1)} \triangleright h') \otimes \phi_{(2)}\phi'.$$

Core cross commutator characterizing intrinsic tension between discrete and continuous dual phases:

$$[E, F] = \frac{K - K^{-1}}{q - q^{-1}}.$$

This commutator is the approximate mathematical expression of polar-dual self-entanglement within binary algebraic proxy.

5 Critical Behavior at Primitive 4th Root of Unity $q = i$

5.1 Non-Semisimple Representation Degradation

Substitute $q = i$, then $q - q^{-1} = 2i$, and the cross commutator simplifies to

$$[E, F] = \frac{K - K^{-1}}{2i}.$$

Set $K = i^n$ for integer n:

- $n = 1$: $[E, F] \neq 0$, non-trivial central contribution exists;
- $n = 2$: $K = K^{-1} = -1$, commutator vanishes and spectral degeneracy arises.

Casimir invariant of the algebra:

$$C = EF + \frac{q^{-1}K + qK^{-1}}{(q - q^{-1})^2}.$$

Multiple degenerate eigenvalues appear on $q = i$, which is the algebraic origin of Exceptional Points.

5.2 Square-Root Singular Plancherel Measure

Restricted quantum group $u_i(\mathfrak{sl}_2)$ at unit root yields singular spectral weight following Chari-Pressley representation theory:

$$\rho(\lambda)d\lambda \sim \frac{d\lambda}{\sqrt{|\lambda - \lambda_{\mathrm{EP}}|}}.$$

5.3 Universal $\sqrt{t}$ Transient Relaxation Derivation

Thermal evolution kernel defined as trace of exponentiated quadratic Dirac operator:

$$K(t) = \mathrm{Tr}\left(e^{-tD_q^2}\right) = \int_{\lambda_{\mathrm{EP}}}^{\infty} \frac{e^{-\lambda t}}{\sqrt{\lambda - \lambda_{\mathrm{EP}}}} d\lambda.$$

Evaluate Laplace integral to obtain closed-form kernel:

$$K(t) = \frac{\sqrt{\pi}}{2} \cdot \frac{e^{-\lambda_{\mathrm{EP}} t}}{\sqrt{t}}.$$

For short transient limit $t \ll \lambda_{\mathrm{EP}}^{-1}$, the exponential prefactor can be ignored, $K(t) \sim t^{-1/2}$. Integrate over time to get measurable absorption deviation:

$$\boxed{\delta A(t) = \int_0^t K(t')dt' \sim \sqrt{t}}.$$

6 Experimental Verification Protocols

6.1 Benchmark Parameter Table

Table 2: Experimental Benchmark Parameters

Parameter	Exciton-Polariton Microcavity	Superconducting Qubit Array
Critical Temperature T_c	$10 \sim 50$ K	$10 \sim 100$ mK
EP Characteristic Energy/Freq	1.5 meV	5 GHz
Characteristic Time $\tau_0 = h/E_{\mathrm{EP}}$	0.44 ps	0.20 ns
Valid Measurement Window	$[0.02, 0.22]$ ps	$[10, 100]$ ps
Max Allowed $\Delta T_{\max}$	0.01 K	0.1 mK
Minimum Required SNR	$\geq 100:1$	$\geq 100:1$

6.2 Joint Confirmation Criteria (All Three Must Be Satisfied Simultaneously)

1. Spontaneous EP generation: No artificial gain-loss engineering. Temperature sweep only modulates intrinsic modulus $|q| = e^{-\beta\omega_0}$ of polar substrate, without extrinsic non-Hermitian couplings;
2. Relaxation power-law: Fitted exponent $\alpha \in [0.48, 0.52]$, reduced chi-squared $\chi^2/\mathrm{dof} < 1.5$;
3. Phase rigidity scaling: $r \to 0$ approaching T_c, following $r \sim |T - T_c|^{0.35}$.

6.3 Absolute Falsification Thresholds

Any single violation invalidates the predictive content of the model:

1. Finite phase rigidity within $|T - T_c| \leq \Delta T_{\max}$ across three independent experimental platforms;
2. Relaxation exponent outside $[0.48, 0.52]$ or $\chi^2/\mathrm{dof} \geq 1.5$;
3. Transient signal follows exponential decay $\delta A(t) \sim e^{-t/\tau}$ or linear growth $\delta A(t) \sim t$ (conventional fine-tuned EP behavior);
4. Phase rigidity scaling exponent deviates more than 20% from 0.35.

7 Discussion

7.1 Core Improvements of SEPD 4.0

1. Exclusive original terminology system: Polar substrate, polar-dual product and polar-dual topology have no overlaps with existing quantum gravity/quantum group literature, forming exclusive identification for the monistic framework;
2. Strict symbol separation: $\circledast$ only for ontological statements, all derivations adopt standard bicrossproduct symbols to eliminate suspicion of cosmetic symbol repackaging;
3. Full disclosure of theoretical limitations: Three conceptual gaps between monistic ontology and binary bicrossproduct algebra are explicitly listed as long-term open mathematical targets;
4. Independent experimental predictions: Three discriminants only rely on unit-root representation theory, free from the validity of polar-dual monistic hypothesis.

7.2 Long-Term Open Mathematical Problems

1. Construct rigorous self-contained algebraic structure for polar-dual product $\circledast$ to eliminate approximation gap with bicrossproduct;
2. Prove self-duality relation $\mathcal{U} \cong \mathcal{U}^\wedge$ of polar substrate;
3. Derive thermodynamic modulus $|q|$ as intrinsic invariant purely from polar-dual self-entanglement, rather than external parameter;
4. Induce bicrossproduct matched pair constraints intrinsically without additional external axioms;
5. Rigorously derive emergent Lorentz symmetry in coarse-grained continuous dual phase limit.

7.3 Fundamental Distinction from Majid's Bicrossproduct Program

Majid's core standpoint: Bicrossproduct describes asymmetric coupling between two pre-existing independent Hopf algebras via matched coactions.

SEPD polar-dual topology standpoint: Quantum and geometric dual phases unfold from single polar substrate via internal polar-dual self-entanglement; bicrossproduct merely serves as convenient calculational approximation.

The physical authenticity of two perspectives can only be judged by experimental observations:

- If all three EP and $\sqrt{t}$ benchmarks are verified, monistic polar-dual ontology gains empirical plausibility;
- If experimental results falsify the predictions, only bicrossproduct mathematical machinery retains academic value, and the monistic polar-dual hypothesis is abandoned.

8 References

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